NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES

DEPARTMENT OF NATURAL AND APPLIED SCIENCES

QUALIFICATION : BACHELOR OF SCIENCE	
QUALIFICATION CODE: 07BOSC	**LEVEL:** 7
COURSE CODE: QPH 702S	**COURSE NAME:** QUANTUM PHYSICS
SESSION: NOVEMBER 2022	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER(S)	**Prof Dipti R. Sahu**
MODERATOR:	**Prof Vijaya S. Vallabhapurapu**

INSTRUCTIONS

1. Answer any **Five** questions.
2. Write clearly and neatly.
3. Number the answers clearly.

PERMISSIBLE MATERIALS

Non-programmable Calculators

THIS QUESTION PAPER CONSISTS OF 4 PAGES (Including this front page)

Question 1 [20]

1.1 List with reason, three properties of a valid wave of a bounded state. (3)

1.2 Replace the following classical mechanical expressions with their corresponding quantum mechanical operators. (6)

a. K.E. = ½ mv^2 in three-dimensional space.

b. p = mv, a three-dimensional cartesian vector.

c. y-component of angular momentum: $L_y = zp_x - xp_z$.

1.3. How to describe a system in quantum mechanics? (4)

1.4 For a particle moving freely along the x-axis, show that the Heisenberg uncertainty principle can be written in the alternative form: $\Delta\lambda\, \Delta x \geq \lambda^2 / 4\pi$ where Δx is the uncertainty in position of the particle and Δλ is the simultaneous uncertainty in the de Broglie wavelength. (5)

1.5 What is the significance of wave packet (2)

Question 2 [20]

2.1 Consider a one-dimensional particle which is confined within the region $0 \leq x \leq a$ and whose wave function is $\psi(x, t) = \sin(\pi x/a) \exp(-i\omega t)$.

(a) Find the potential V(x). (5)

(b) Calculate the probability of finding the particle in the interval $a/4 \leq x \leq 3a/4$. (5)

2.2 Consider the one-dimensional wave function (10)

$$\Psi(x) = A\left({}^{x}/_{x_0}\right)^n e^{-x/x_0}$$

where A, n and x_0 are constants. Using Schrodinger's equation, find the potential *V(x)* and energy *E* for which this wave function is an eigenfunction. (Assume that as $x \to \infty$, $V(x) \to 0$).

Question 3 [20]

3.1 The wavefunction of a particle moving in the x-dimension is

$$\psi(x) = \begin{cases} Nx(L-x) & 0 < x < L \\ 0 & \text{elsewhere} \end{cases}$$

3.1.1 Normalize the wavefunction (4)

3.1.2 Determine the expectation value of x (4)

3.2 Evaluate the probability current density of the wavefunction, (2)

$$\Psi(x) = 5\exp(-3ix)$$

3.3 The potential function V(x) of the problem is given by (10)

$$V(x) = \begin{cases} V_o & x > 0 \\ 0 & x < 0 \end{cases}$$

where V_o is constant potential energy.

Find the wave function for $E < V_o$ where E is the incident particle energy and interpret the results.

Question 4 **[20]**

4.1 Obtain the spin matrix S_2 for spin s = $\frac{3}{2}$ particle using the eigenstates of S^2 as the basis (10)

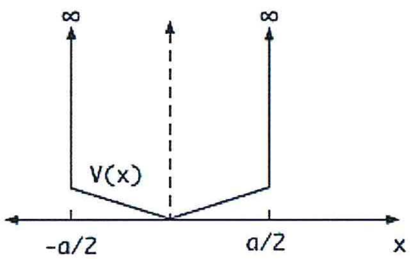
4.2 Evaluate the commutation of L_2, L_3. (5)

4.3 Consider a system which is initially in the state (5)

$\Psi(\Theta, \phi) = \frac{1}{\sqrt{5}} Y_{1,-1}((\Theta, \phi) + \sqrt{\frac{3}{5}} Y_{1,0}(\Theta, \phi) + \frac{1}{\sqrt{5}} Y_{1,1}(\Theta, \phi)$, Find $< \psi | L_+ | \psi >$

Question 5 **[20]**

5.1 Consider an infinite well for which the bottom is not flat, as sketched here. If the slope is small, the potential $V = \varepsilon\, |x| / a$ may be considered as a perturbation on the square-well potential over $-a/2 \leq x \leq a/2$. (5)



Calculate the ground-state energy, correct to first order in perturbation theory. Given Ground state of box size a : $\psi_0 = \sqrt{(2/a)}\ \text{Cos}\,\frac{\pi x}{a}$, Ground state energy $E_0 = \frac{h^2\pi^2}{2ma^2}$

5.2 The wave function of the ground state of hydrogen has the form. (5)

$$\Psi_{100} = \frac{1}{\sqrt{\pi r_0^3}} e^{\frac{-r}{r_0}}$$

Find the probability of finding the electron in a volume dV around a given point.

5.3 Evaluate the constant B in the hydrogen-like wave function (10)

$$\Psi(r, \Theta, \phi) = B\, r^2 \sin^2\Theta\, e^{2i\varphi} \exp\left(-\frac{3Zr}{3a_0}\right)$$

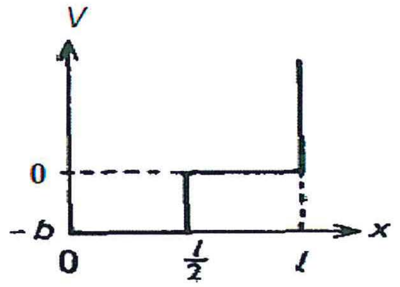
Question 6 **[20]**

6.1 The wavefunction of a state of harmonic oscillator is given by: (10)

$$\Phi(x) = \left(\frac{m\omega}{64\pi\hbar}\right)^{\frac{1}{4}} \left(\frac{4m\omega}{\hbar}x^2 - 2\right) \quad \exp(-mwx^2/2\hbar) \qquad ; -\infty < x < \infty$$

Obtain the corresponding energy of the state.

6.2 A particle moves in a one-dimensional box with a small potential dip (10)



$V = \infty$ for $x<0$ and $x > l$
$V = -b$ for $0 < l < (l/2)\, l$
$V = 0$ for $(l/2)\, l < x < l$

Treat the potential dip as a perturbation to a regular rigid box ($V = \infty$ for $x < 0$ and $x > l$, $V = 0$ for $0 < x < l$). Find the first order energy of the ground state. The ground state energy and wavefunction is given by $E^0 = \frac{\pi^2\hbar^2}{2ml^2}$, $\psi^0(x) = \sqrt{\frac{2}{l}} \sin\frac{\pi x}{l}$

....END..

Useful Standard Integral

$$\int_{-\infty}^{\infty} e^{-y^2} dy = \sqrt{\pi}$$

$$\int_{-\infty}^{\infty} y^n e^{-y^2} dy = \frac{\sqrt{\pi}}{n}; \quad n \text{ even}$$
$$0; \quad n \text{ odd}$$

$$\int_{-\infty}^{\infty} e^{-\alpha y^2} e^{-\beta y} dy = \left(\frac{\pi}{\alpha}\right)^{\frac{1}{2}} e^{\frac{\beta^2}{4\alpha}}$$

$$\int_0^{\infty} x^n e^{-x} dx = n!$$